Physics of Buildings

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Adapted from Chapters 11 and 12:

Physics of Societal Issues:

Calculations on National Security, Environment and Energy

(Springer, 2007)

Energy in Buildings

- Linearized Heat Transfer
- Free Temperature
- Scaling Model of a Cubic Building
- Passive Solar Heating
- Thermal Flywheel House

Linearized Heat Transfer

DC Circuits: $V = I R$, similar to

Heat Conduction: $dQ/dt = UA\Delta T = (1/R)A\Delta T$

$U \Rightarrow$ Btu/ft^2-hr-^{o}F = 1 Art (Henry Kelly)
(W/m^2-^{o}C)

Steady state heat transfer is similar to DC circuits

$V =$	I	R
$\Delta T =$	dQ/dt	(R/A)

This ignores heating up/down, important in CA,
…….less so in Chicago

mass similar to capacitance, $V = Q/C$ and $\Delta T = \Delta Q\ (1/mc)$

mass and R are continuous media, a leaky capacitor

no heat inductance, $V = L\ (dI/dt)$, needs d^2Q/dt^2

Radiation: $dQ/dt_{radiation} = \sigma A(\varepsilon_i T_i^4 - \varepsilon_o T_o^4) = U_{radiation} A \Delta T$

$U_{radiation} = 4\varepsilon\sigma T_1^3 = (4\varepsilon)(5.7 \times 10^{-8})(293\ K)^3 = 5.7\ \varepsilon\ SI = 1\ \varepsilon\ UK$

Convection = f(geometry, wind, surface)

$$dQ/dt_{convection} = hA(\Delta T)^{5/4} = (h\Delta T^{1/4})A\Delta T = U_{convection} A \Delta T$$

$$U_{convection} \approx U_{radaiation}$$

Free Temperature

Old House, 40x40= 1600 ft^2 x 10 ft ceiling, sum of UA's

walls(R5),	1600-400 ft^2 x U0.2	= 240
ceil/floor(R10),	1600 ft^2 x U0.1 x 1.5	= 240
window(R1),	400 ft^2 x U1	= 400

Lossiness = ΣUA =

240 + 240 + 400 + 30% infil. =1150 Btu/hr-^{o}F

Free temperature rises with internal heat of 1 kW (3500 Btu/hr)

$$\Delta T_{free} = (dQ/dt)/\Sigma UA = 3500/1150 = 3^oF$$

$$T_{balance} = T_{thermostat} - \Delta T_{free} = 68\ ^oF - 3\ ^oF = 65\ ^oF$$

with 2 kW (x 2) and 200 Btu/hr-^{o}F (x 1/5): $\Delta T_{free} = 35^oF$

No heating needed until 35^{o}F

10% heating needed at 0^{o}F

[20% loss rate][70 – 35 – 0^{o}F]/[70 – 0 ^{o}F] = 0.2 x 0.5 = 0.1

Annual Energy Use

Large buildings are driven more by internal heat loads.

Small buildings are driven by climate and their skins.

Degree days are less relevant to CA, as compared to Chicago

Degree Days: Annual Heat Loss $Q = \Sigma\Sigma\ (dQ/dt)\ \Delta t$

$$Q = \Sigma^{n}\ U_j A_j\ \Sigma^{8766}\ (T_{\text{base}} - T_{\text{outside}})_i(1\text{ hour})$$

degree-hours per year (*dh/yr*):

$dh/yr = \Sigma^{8766} (T_{\text{base}} - T_{\text{outside}})_i(1 \text{ hour})$

degree-days per year (*dd/yr*):

$$dd/yr = \Sigma^{8766} (65°\text{F} - T_{\text{outside}})_i(1 \text{ hour})/24$$

$Q_{\text{needed}} = (dd/yr)(24 \text{ hr/day})(1/\text{efficiency}) \, \Sigma^n U_j A_j$

Chicago (6200 dd), lossiness improved to 600

$Q = (6200 \text{ dd/y})(24)(3/2)(600) = 1.3 \times 10^8 \text{ BTU/y} = 20 \text{ bbl/y}$

Infiltration Energy Loss

$$dQ/dt_{\text{infil}} = (dm/dt)\ c\ \Delta T$$

dm/dt = infiltration rate of air mass, c = specific heat of air

R_{ACH} = 1/t ach (air exchanges/hour)
100% of interior air mass exhausted in $t_{\text{residence}}$ hours.

$$dQ/dt_{\text{infil}} = (V\rho)\ R_{\text{ACH}}\ c\Delta T$$

$V\rho$ = mass of interior air (volume x density).

Annual heat energy <u>needed</u> over the year is

$$dQ/dt_{\text{infil}} = (V\rho)\, R_{\text{ACH}}\, c\, (dd/yr)\, (24\ \text{hr/day}) / \eta$$

$V = 2.5\ \text{m} \times 140\ \text{m}^2$ ($8.2\ \text{ft} \times 1500\ \text{ft}^2$)
$R_{\text{ACH}} = 0.8$ ach
$\rho = 1.3\ \text{kg/m}^3$ ($0.0735\ \text{lb/ft}^3$)
$c = 1004\ \text{J/kg-}^{\text{o}}\text{C}$ ($0.24\ \text{Btu/lb-}^{\circ}\text{F}$)
dd/yr = 2800^{o}C-day/yr (5000^{o}F-day/yr)
$\eta = 2/3$

$dQ/dt = (140 \times 2.5\ \text{m}^3)(0.8\ \text{ach})(1.3\ \text{kg/m}^3)$
$(1004\ \text{J/kg-}^{\text{o}}\text{C})(24\ \text{h/d})(2800\ ^{\text{o}}\text{C-d/yr})$
$= 3.7 \times 10^{10}$ J = 35 MBtu/yr = 5 bbl/yr

Energy loss proportional to ach **α ach**
Bad health effects proportional $t_{\text{residence}}$ **$t_{\text{residence}}\ \alpha$ 1/ach**

Scaling Model of a Cubic Building

$dQ/dt]_{loss} = \quad UA\,\Delta T = KL^2\Delta T$

$dQ/dT]_{gain} = FnL^2 = FL^3/H \; = GL^3 \qquad\qquad (n = L/H)$

$F \quad = \quad 66 \quad W/m^2$ ($6\ W/ft^2$), $H = 3$ m ➔ $G = 22\ W/m^3$

$dQ/dt]_{gain} – dQ/dt]_{loss} = GL^3 – KL^2\Delta T_{free}$

$\Delta T_{free} = (G/K)L$

Large Building:

$1.5L^2$	ceilings [R15, R_{SI}2.62], floors 50% of ceilings
$0.7 \times 4L^2$	70% walls [R6.5, R_{SI}1.14]
$0.3 \times 4L^2$	30% windows [R1, R_{SI}0.16]
x 1.3	infiltration

$$K = (1.3)[1.5/2.62 + 0.7(4)/1.14 + 0.3(4)/0.16] = 14$$

$$\Delta T_{free} = (G/K)L = (22/14)L = 1.6\ L$$

L=10 m (33 ft), = 16°C (28°F) [skin dominated]

Multiple Savings

$dQ/dt]_{net} = dQ/dt]_{loss} - dQ/dt]_{gain} = KL^2[\Delta T - \Delta T_{free}] =$

$dQ/dt]_{net} = KL^2[\Delta T - (G/K)L]$

Reduced conductivity K saves by

- multiplicative KL^2
- subtractive $\Delta T_{free} = (G/K)L$
- degree-day distribution (some days save 100%, other days f%)
- Store day-time heat for cool evenings
- Save evening coolth for daytime air-conditioning
- infiltration can then dominate, use air-to-air heat exchangers
- "heat with two cats fighting" [Lovins], but economics enters

Passive Solar Heating

Insulate before you *insolate*.

Glass plus Mass prevents you

from freezing your ….!

H to He: $\Delta m/m = (4 \times 1.0078 - 4.0026)/(4 \times 1.0078) = 0.7\%$

$\Delta E_{sun} = \Delta mc^2 = (0.0071)(2.0 \times 10^{30}\ \text{kg}/\mathbf{10})(3 \times 10^8\ \text{m/s})^2 = 1.4 \times 10^{44}\ \text{J}$

Solar average power

$$\mathbf{P} = \Delta E/\Delta t = (1.4 \times 10^{44}\ \text{J}/10^{10}\ \text{y}) = 3.9 \times 10^{26}\ \text{W}$$

Solar flux at Earth (1.37 kW/m²)

$S_o = P/4\pi(1\ \text{AU})^2 = (4.4 \times 10^{44}\ \text{W})/(4\pi)(1.5 \times 10^{11}\ \text{m})^2$
$= 1.6\ \text{kW/m}^2$

Solar flux absorbed, ΔS, in small air mass Δm:

$$\Delta S = -\lambda S \ \Delta m,$$

where λ is absorption constant. This integrates to

$$S_1 = S_o \ e^{-\lambda m}.$$

Air mass increases with angle θ from zenith

$$m = Nm_o = m_o \sec(\theta)$$

where m_o is air mass at $\theta = 0^o$. Solar flux at angle θ,

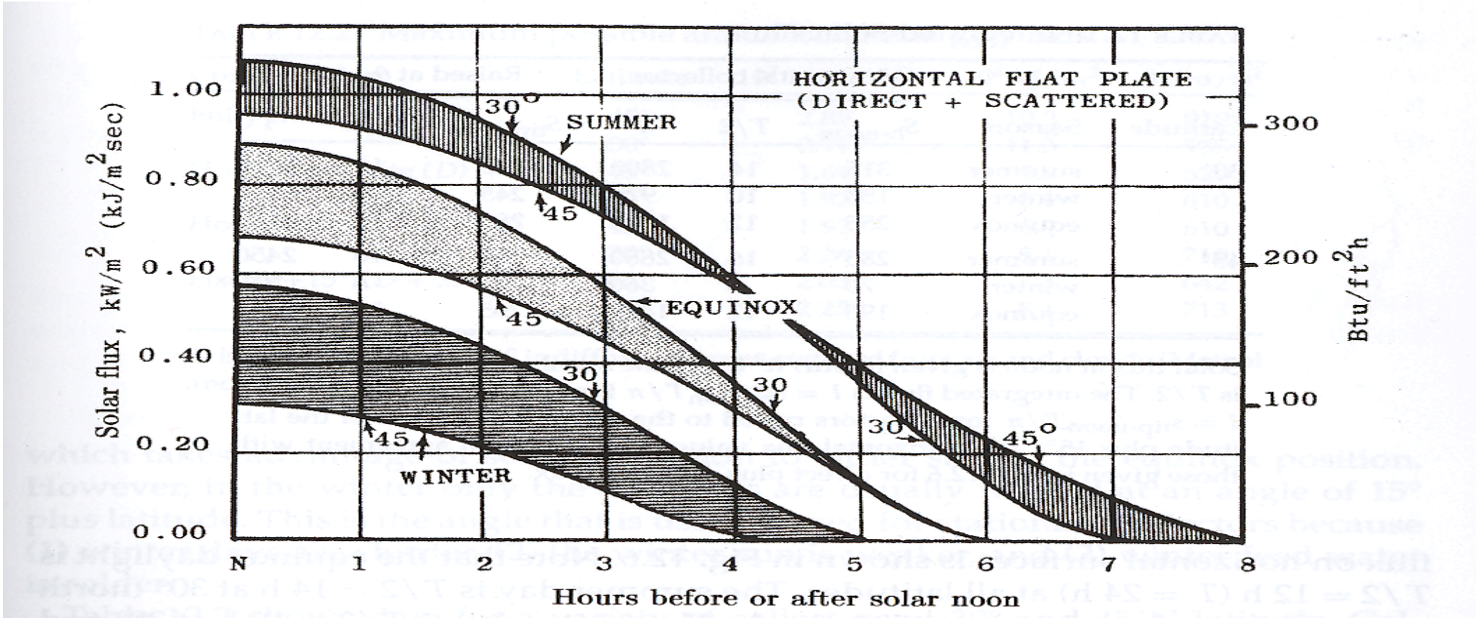
$$S_1 = S_o \exp[-\lambda m_o \sec(\theta)]$$

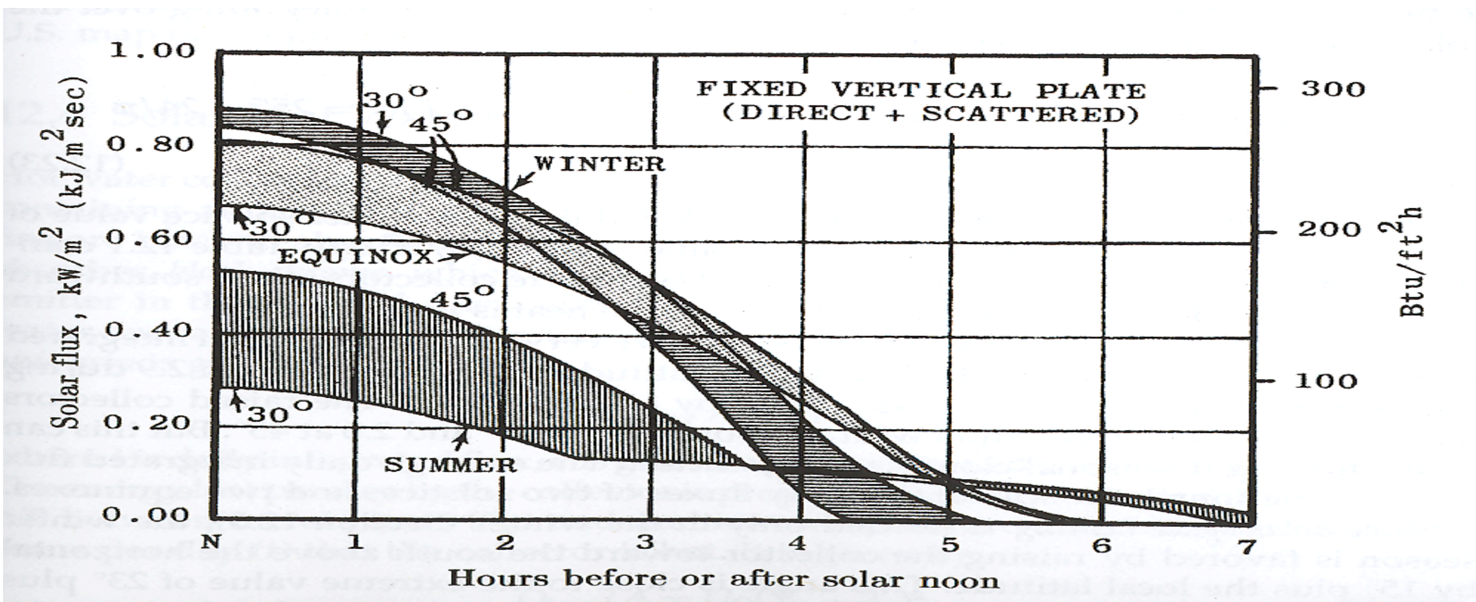
λm_o determined from flux above atmosphere (S_o = 1370 W/m^2) and at Earth's surface (S_1 = 970 W/m^2) when sun in zenith:

$$S_1 = 970\ \mathrm{W/m^2} = 1370\ \mathrm{W/m^2}\ \exp(-\lambda m_o).$$

This gives λm_o = 0.33 and solar flux at sea level ,

$$S_1 = S_o\, e^{-0.33\sec(\theta)} = S_o\, e^{-1/3\cos(\theta)}$$


HORIZONTAL FLAT PLATE
(DIRECT + SCATTERED)
SUMMER
EQUINOX
WINTER
30°
45
30
Solar flux, kW/m² (kJ/m² sec)
Btu/ft²h
Hours before or after solar noon
N
1.00
0.80
0.60
0.40
0.20
0.00
300
200
100


FIXED VERTICAL PLATE
(DIRECT + SCATTERED)
30°
45°
WINTER
EQUINOX
SUMMER
Solar flux, kW/m² (kJ/m² sec)
Btu/ft²h
Hours before or after solar noon
N
1.00
0.80
0.60
0.40
0.20
0.00
300
200
100

SLO vertical window at noon on winter solstice ($\theta = 34^o + 23^o = 57^o$)

$$S_{Vo} = [S_o \sin(57^o)]\ [e^{-1/3\cos(57)}] = [435][0.84][0.542]$$

$$= 200\ \text{Btu/ft}^2\text{-hr}$$

Integrated solar flux over a day:

$$I = \int_0^{T/2} S_V\, dt = \int_0^{T/2} S_{Vo} \sin(2\pi t/T)\, dt = S_{Vo} T/\pi$$

$$= 200 \times 20/\pi = 1280\ \text{Btu/ft}^2\text{-d}$$

Winter window gains and losses: (San Luis Obispo)

single-glaze [$U = 1$ Btu/ft^{2}-hr]

50 °F outside temperature

90% transmission through

flux, south-facing,

$S_V = [270 \text{ Btu/ft}^2\text{hr}] \sin(2\pi t/T)$, $T/2 = 10$ hour

Heat loss:

$Q_{loss}/A = U\,\Delta T\,\Delta t = (1)(65\ °F - 50\ °F)(24 \text{ hours}) =$

$= 400$ Btu/ft^{2}-d

Solar gain:

$Q_{gain}/A = 0.9I = (0.9)(1300\ Btu/ft^2\text{-}d) = 1150\ Btu/ft^2\text{-}d.$

$Q_{gain}/Q_{loss} = 1150/400 = 3$

Improvements:

- drapes or R-11 blinds at night
- double-glaze, low-E windows (R4)
- R10 Windows

Thermal Flywheel House

A tank of water 25 cm thick is “optimal.”

Ignore small temperature variations over the volume .

$$Q = mC\,\Delta T,$$

- m is water mass
- C is specific heat
- ΔT is temperature difference between the tank and room

Heat loss rate from tank volume:

$$dQ/dt = mC\ dT/dt$$

Heat-loss rate from surface:

$$dQ/dt = AU_{total}\Delta T$$

- A = tank area (m^2)
- $U_{total} = U_{convection} + U_{radiation} = 12\ W/m^2$
- $\Delta T = T_{barrel} - T_{room}$ = T above room temperature

Equating surface loss rate to volume loss rate:

$dQ/dt = AU_{total}T = -mC\, dT/dt,$

gives: $T = T_o e^{-t/\tau}$ and $\tau = mC/AU_{total}$

A 25-cm thick tank on a square meter basis:

$\tau = mC/AU_{total}$

$= (250\text{ kg})(4200\text{ J/kg-}^{o}\text{C})/(2\text{ m}^2)(12\text{ W/m}^2\text{-}^{o}\text{C}) = 12\text{ hr}$