Paleoclimatic warming increased carbon dioxide concentrations

D. M. Lemoine

Received 6 July 2010; revised 22 August 2010; accepted 13 September 2010; published 30 November 2010.

[1] If climate–carbon feedbacks are positive, then warming causes changes in carbon dioxide (CO₂) sources and sinks that increase CO₂ concentrations and create further warming. Previous work using paleoclimatic reconstructions has not disentangled the causal effect of interest from the effects of reverse causality and autocorrelation. The response of CO₂ to variations in orbital forcing over the past 800,000 years suggests that millennial-scale climate–carbon feedbacks are significantly positive and significantly greater than century-scale feedbacks. Feedbacks are also significantly greater on 100 year time scales than on 50 year time scales over the past 1500 years. Posterior probability distributions implied by coupled models’ predictions and by these paleoclimatic results give a mean of 0.03 for the nondimensional climate–carbon feedback factor and a 90% chance of its being between −0.04 and 0.09. The 70% chance that climate–carbon feedbacks are positive implies that temperature change projections tend to underestimate an emission path’s consequences if they do not allow the carbon cycle to respond to changing temperatures.


1. Introduction

[2] Climate–carbon (or carbon cycle) feedbacks control how carbon dioxide (CO₂) concentrations respond to changing temperatures [Friedlingstein et al., 2006; Gregory et al., 2009]. Positive feedbacks indicate that increased surface temperatures cause changes in CO₂ sources and sinks that in turn further increase surface temperatures [Cox et al., 2000; Heimann and Reichstein, 2008]. Because other climate change feedbacks are thought to be positive on net [Bony et al., 2006; Soden et al., 2008], and because feedbacks add linearly but impact temperature nonlinearly [Torn and Harte, 2006; Roe and Baker, 2007; Roe, 2009], constraining the range of climate–carbon feedbacks is important for constraining temperature change projections and for climate risk assessments [Plattner et al., 2008; Huntingford et al., 2009]. However, while models that couple the carbon cycle and the climate system can provide some insight into the possible magnitude of these feedbacks, the number and complexity of the interlinked processes restrict the amount of information that can be gleaned from models alone [Lemoine, 2010].

[3] Estimates from paleoclimatic data can provide an alternate source of information about the scale of feedbacks that may operate under anthropogenic warming. While differences in boundary conditions and in the type of forcing mean that paleoclimatic data are unlikely to correctly describe the Earth system’s response to ongoing anthropogenic greenhouse gas forcing, their biases in the anthropogenic application might be largely uncorrelated with those impacting coupled models’ predictions [Lemoine, 2010]. Paleoclimatic estimates can therefore complement models’ predictions in the construction of a probability distribution for climate–carbon feedbacks.

[4] This paper estimates climate–carbon feedback strength over past ice age cycles and over the past two millennia. It uses changes in insolation due to orbital variations to identify the response of atmospheric CO₂ concentrations to changes in temperature over the previous 800,000 years. The results indicate that climate–carbon feedbacks were probably positive over past ice ages and over the past two millennia. The magnitude depends on the time scale of interest but, over millennial time scales, is comparable to coupled models’ predictions of the carbon cycle’s response to anthropogenic greenhouse gas forcing. The temperature change produced by a given emission path is therefore probably greater than suggested by climate sensitivity metrics that do not allow the carbon cycle to respond to changing temperatures.

2. Assessing Feedback Strength

[5] The equilibrium temperature change ΔT due to a change in radiative forcing can be represented as

\[ \Delta T = \frac{\lambda_0 \Delta R_f}{1 - \sum_{k=1}^{\infty} \epsilon_k \lambda_0} = \frac{\lambda_0 \Delta R_f}{1 - F}, \]

where \( \lambda_0 \) is the temperature change per unit of radiative forcing in the reference system upon which feedbacks operate, \( \Delta R_f \) is the exogenous change in radiative forcing...
produced by increased greenhouse gas concentrations, and nondimensional \( f_k \equiv c_k \lambda_0 \) gives the influence of feedback process \( k \) [Roe, 2009]. This representation assumes that feedback processes are linear over the relevant temperature range and are defined so that they interact only through their effects on temperature. When positive, \( f_k \) may be interpreted as the fraction of total warming due to feedback process \( k \).

[6] Each feedback factor \( f_k \) can be decomposed into the product of the total change in climate field \( \alpha_k \) due to a unit change in temperature and the change in radiative forcing due to a unit change in climate field \( \alpha_k \) when other climate fields are held fixed [Roe, 2009]. In the case of climate-carbon feedbacks \( f_{cc} \) affecting \( CO_2 \) concentrations, this gives

\[
f_{cc} \equiv f_k = \lambda_0 \left( \frac{\partial R}{\partial \ln CO_2} \right)_{\alpha_{fix}} \frac{d \ln CO_2}{dT} ,
\]

(2)

where climate-carbon feedbacks are feedback process \( k \). \( CO_2 \) concentrations are represented by their log because radiative forcing increases approximately linearly with the log of \( CO_2 \), yielding \( \frac{\partial R}{\partial \ln CO_2} \) \( = 5.35 \) W m\(^{-2} \) (ln ppm\(^{-1} \)) [Ramaswamy et al., 2001, Table 6.2]. \( \lambda_0 \) is approximately \( 0.315 \) K (W m\(^{-2} \) \( \cdot \) K\(^{-1} \)) [Soden et al., 2008]. Estimating the climate-carbon feedback factor \( f_{cc} \) therefore primarily requires estimating \( \psi \equiv d \ln CO_2/dT \), or the effect of a unit of temperature change on \( CO_2 \) concentrations. Coupled climate-carbon cycle models have predicted this term [Friedlingstein et al., 2003, 2006; Cadule et al., 2009], but these models provide limited information because they only include a subset of known carbon cycle processes and are vulnerable to the possibility of shared model biases [Luo, 2007; Tebaldi and Knutti, 2007; Lemoine, 2010].

[7] Paleoclimatic estimates can provide an important additional source of information with biases largely independent of models’ shared biases, but empirical estimation is complicated by the degree to which Earth system components are intertwined, by the incompleteness of climatic records, and by the inability to run full-scale controlled experiments. Four studies have attempted to constrain climate-carbon feedbacks from temperature and \( CO_2 \) reconstructions. Scheffer et al. [2006] considered the last millennium’s Little Ice Age (LIA), and Torn and Harte [2006] used the last 360,000 years as recorded by the Vostok ice core. Frank et al. [2010] estimated the response of \( CO_2 \) to temperature for three time periods in the past millennium. An ensemble of temperature and \( CO_2 \) reconstructions produced a frequency distribution for \( \psi \). This distribution may be interpreted as a probability distribution for \( \psi \) if one assumes that the reconstructions properly sample the space of possible worlds. Finally, Cox and Jones [2008] constrained climate-carbon feedback strength by determining which values are consistent with the output of coupled climate-carbon cycle models run using twentieth century data, with the results of matching coupled models to observed interannual variability, and with a LIA analysis closely related to that of Scheffer et al. [2006].

[8] Crucially, these four studies rely on univariate regressions of \( CO_2 \) on temperature that may contain biases from reverse causality and autocorrelation (Appendix A). A univariate regression cannot disentangle whether high \( CO_2 \) levels accompany high temperatures because higher \( CO_2 \) causes higher temperatures, because higher temperatures cause higher \( CO_2 \), or because they are each being driven by, for instance, previous periods’ \( CO_2 \) and temperature. Because feedback estimation is concerned with the response of \( CO_2 \) to an exogenous increase in temperature, it is important that paleoclimatic studies isolate the response of \( CO_2 \) to temperature from the more general correlation estimated by a univariate regression. The present study seeks to isolate the causal effect of temperature on \( CO_2 \) by looking at the response of \( CO_2 \) to variations in temperature that were unlikely to be caused by variations in \( CO_2 \).

3. Methods: Estimated Equations

[9] The present study estimates climate-carbon feedbacks over four time scales: millennia, centuries, 100 years, and 50 years. It seeks to generate estimates that are free of simultaneous equations (or reverse causality) bias and omitted variables bias. First, it aims to avoid simultaneous equations bias by using orbital forcing as an instrument for temperature over the longer time scales (Appendix A). A good instrument is correlated with temperature but only affects the coeval \( CO_2 \) concentration through its effect on temperature. In other words, using this instrument isolates a “good” portion of the variation in temperature—a portion that is believed not to be caused by changes in \( CO_2 \)—and ignores the rest. A good instrument avoids the problem of imputing the causal effect of temperature on \( CO_2 \) from data that actually reflects the greenhouse effect of \( CO_2 \) on temperature.

[10] The key hypotheses for the validity of an orbital forcing instrument are that: (1) changes in orbital forcing cause changes in temperature but (2) do not affect \( CO_2 \) levels except through their effect on temperature. If these hypotheses hold, then we can replace the actual temperature record with one predicted from orbital forcing data and believe that any remaining correlation with the \( CO_2 \) record is due to the effect of temperature on \( CO_2 \). The first hypothesis is supported by the Milankovitch theory of glacial cycles, according to which summer insolation in the Northern Hemisphere’s high latitudes controls both hemispheres’ temperature on millennial time scales [Milankovitch, 1941; Hays et al., 1976; Berger, 1992]. Variations in summer insolation might have this effect because nonlinearities in the climate system can amplify the direct effect on ice sheets and snow accumulation. Importantly for the choice of which insolation time series to use, some have instead argued that the true trigger for deglaciation is the timing of summer insolation in the Northern Hemisphere [Hansen et al., 2007] or that Antarctic temperatures are more tightly controlled by the duration of the local (Southern Hemisphere) summer [Huybers and Denton, 2008]. While the hypotheses are difficult to distinguish empirically [Huybers, 2009] and the true mechanism may be more complex [Wolff et al., 2009], recent evidence does support a Northern Hemisphere trigger for Antarctic temperatures [Kawamura et al., 2007; Cheng et al., 2009]. Further, several recent studies [Petit et al., 1999; Jouzel et al., 2007; Kawamura et al., 2007] used high-latitude summer insolation in the Northern Hemisphere as an indicator of orbital forcing, and
ice core chronologies sometimes assume a linear response of climate to orbital forcing, whether defined via mid-June insolation at northern high latitudes [Parrenin et al., 2004] or via anticipated periodicity [Salamatin, 2000]. Therefore, given that orbital forcing should affect temperature, the key condition becomes the hypothesis that it does not directly affect the CO₂ concentration. Because orbital forcing affects insolation at the poles and the least at the equator, and because the primary effect at the poles is on snow and ice melt (via temperature), orbital forcing’s effect on the timing and spatial distribution of insolation may not be directly critical for important carbon sources and sinks. Variations in orbital forcing may in fact cause variations in temperature without affecting CO₂ concentrations except through these variations in temperature.

[11] The second source of bias that univariate regressions are exposed to is omitted variables bias produced by correlation of time t temperature and CO₂ with previous temperature and CO₂. If not accounted for, such correlation with past climate states could induce correlation between time t temperature and CO₂ that univariate regressions include in their coefficient estimates. However, this correlation through previous climate states may not be the effect of interest in a feedback application. The present study seeks to minimize omitted variables bias by including lagged covariates in the regression. The estimated model assumes that temperatures and concentrations at times earlier than those included as covariates only affect the temperature and concentration at time t through their effect on the included covariates.

[12] The present study does not eliminate a final source of bias. Measurement error in temperature data may be due to errors in measurement of isotopes, inferences about local temperature from isotope ratios, and inferences about global temperature from local temperature, and in the assignment of relative dates to the recorded temperature and CO₂. This measurement error tends to push coefficient estimates toward zero (Appendix A). Further, gas diffusion processes mean that each CO₂ observation actually has a distribution of ages and an effective resolution of a few centuries [Spahni et al., 2003], which tends to reduce the variation useful for regression-based estimates. The remaining errors should therefore tend to bias the results toward finding no effect of temperature on CO₂.

[13] The orbital forcing specification estimates the following equation:

\[ C_t = \beta_0 + \sum_{i=1}^{2} \beta_i T_{t-i} + \beta_4 C_{t-1} + \epsilon_t, \]

where \( C_t \) is the log of the CO₂ concentration at time t, \( T_t \) is the temperature at time t, and i is in thousands of years. \( C_{t-1} \) is not included as a covariate because CO₂ concentrations from 2000 years ago should only affect contemporary CO₂ concentrations via their effect on CO₂ concentrations and temperature 1000 years ago. Orbital forcing \( (O_t) \) in W m⁻² instruments for \( T_t \) via the following first-stage regression:

\[ T_t = \gamma_0 + \gamma_1 O_t + \sum_{i=1}^{3} \gamma_{i+1} T_{t-i} + \gamma_4 C_{t-1} + \nu_t. \]

\( O_t \) and \( T_t \) have a correlation coefficient of 0.18, so, as required for valid use as an instrument, variation in orbital forcing is connected to variation in temperature. The estimated covariance matrix uses the Huber-White estimator that is robust to arbitrary heteroskedasticity. Importantly for the applicability of the statistical methods used here, the time series appear to be stationary (augmented Dickey-Fuller tests reject the unit root hypothesis at the \( \alpha = 0.05 \) level), which means that the mean and covariance are not changing over time. It is also important that the error term \( \epsilon_t \) not be serially correlated, because serial correlation may mean that \( \epsilon_t \) is correlated with \( C_{t-1} \) via its correlation with \( \epsilon_{t-1} \), which would violate the assumption of exogeneity of the covariates. We test for such serial correlation in the instrumental variable estimate by using a Cumby-Huizinga test, which fails to reject the null hypothesis of no serial correlation at the \( \alpha = 0.20 \) level. We therefore assume that \( \epsilon_t \) is not serially correlated and that \( C_{t-1} \) is in fact exogenous for \( \epsilon_t \).

[14] The resulting coefficients and covariance matrix enable estimation of feedbacks over two time scales. The feedback factor over a time scale of \( j \) time units is calculated from equation (2) using

\[ \psi_j = \sum_{i=0}^{j} T_{t-i} + \sum_{k=1}^{j} \psi_{j-k} C_{t-k}, \] (5)

where \( C \) and \( T \) variables represent their estimated coefficients and \( j \geq 0 \). \( \psi_j \) is defined recursively, and \( \psi_0 \) is the coefficient on \( T_t \). Thus, \( \beta_i \) gives the effect of \( T_t \) on \( C_t \), which is here labeled the century-scale response, and \( \beta_1 + \beta_2 + \beta_4 \beta_1 \) gives the effect on \( C_t \) of an increase in temperature at time \( t-1 \) that is maintained at time \( t \), which is here labeled the millennial response. Variance and covariance calculations use first-order linear approximations for the \( \psi_{j-k} C_{t-k} \) terms.

[15] The data are an 800 kyr temperature record from the Antarctic EPICA Dome C core with the EDC3 age scale [Jouzel et al., 2007], an 800 kyr composite CO₂ record drawn from that and other cores [Lüthi et al., 2008], and the calculations of Berger [1978] for orbital forcing at 60°N (Figure 1a). The similarity of this temperature record to those of the Vostok and Dome F cores implies that it may be indicative of general conditions over eastern Antarctica [Jouzel et al., 2007], and models suggest that Antarctic temperatures may track global temperatures [Masson-Delmotte et al., 2010]. Figure 2 shows how including lagged variables as covariates alters the temperature–CO₂ relationship and how the instrument isolates a portion of the variation in \( T_t \).

[16] Estimating coefficients in several model specifications assesses the results’ robustness to some types of specification error. In the base case and summer insolation specifications, the temperature and CO₂ data used are the observations closest to the endpoint of each 1000 year interval, while the averaged data specification uses the average of the previous 1000 years’ observations. In the base case and averaged data specifications, the orbital forcing instrument is insolation in mid-June, but the summer insolation specification sums the insolation over June, July, and August.
The estimated model for shorter-term feedbacks is

\[
\Delta C_t = \sum_{i=0}^{k} \beta_{1+i} \Delta T_{t-i} + \sum_{j=1}^{k} \beta_{1+k+j} \Delta C_{t-j} + \epsilon_t, \quad (6)
\]

where \(\Delta\) indicates a first difference (so \(\Delta C_t = C_t - C_{t-1}\)) and where \(k = 11\) when the time step for \(t\) is 10 years while \(k = 5\) when the time step for \(t\) is 25 years. Differencing the data makes it stationary (augmented Dickey-Fuller tests reject the unit root hypothesis at the \(\alpha = 0.10\) level), and Durbin’s alternative test, a standard test for serial correlation in Ordinary Least Squares estimates, fails to reject the null hypothesis of no serial correlation at the \(\alpha = 0.50\) level. We calculate the effect of a 50 year and 100 year maintained increase in temperature from equation (5) using the estimated coefficients and heteroskedasticity–robust covariance matrix.

These subcentury time scale specifications do not instrument for \(T_t\) for two reasons. First, simultaneous equations bias should be small. This is because any unobserved sources of variation in \(C_t\) levels that appear between time \(t-1\) and time \(t\) should be small and may not have enough time to fully affect \(T_t\). Second, despite significant first-stage coefficients, weak instrument tests indicate potential problems with the use of solar activity from Steinilber et al. [2009] and Delaygue and Bard [2010] as an instrument for \(T_t\). Even if simultaneous equations bias is nonzero, it is probably sufficiently small that the ordinary least squares estimate is preferable to estimation with a weak instrument.

4. Results

The orbital forcing specifications indicate that expected millennial-scale climate–carbon feedbacks are probably positive (\(p < 0.001\)), acting to amplify anthropogenic warming (Table 1). Their 95% confidence intervals are in the range of 0.02 to 0.05 (Figure 3), which is comparable to the predictions of the coupled climate–carbon cycle models described by Friedlingstein et al. [2006]. However, in line with the anticipated effects of biases introduced to previous work by reverse causality and autocorrelation, this range is on the low end of previous paleoclimatic estimates. Climate–carbon feedbacks are statistically greater over millennial time scales than over time scales of centuries (\(p < 0.001\)), and for either 10 year or 25 year time steps, climate–carbon feedbacks are statistically greater over 100 year time scales than over 50 year time scales (\(p < 0.001\)). Each first-stage regression produces a coefficient on the orbital forcing instrument that is significantly different from 0 (\(p < 0.001\)), and heteroskedasticity–robust Kleibergen-Paap F statistics greater than 15 confirm that the orbital forcing instrument should not pose weak instrument problems.

Most coefficient estimates are fairly stable across orbital forcing specifications and have the expected signs, indicating that the general model is robust to the specifications considered here (Table 2). Both millennial and century-scale feedback estimates are also relatively stable over different 200 kyr sections of the data sets, with the main variations correlated with variations in the strength of the...
instrument (Figure 4). The paper’s main findings therefore should not be highly sensitive to the choice of time period.

Univariate regressions and a noninstrumented multivariate regression help assess the possible importance of omitted variables bias and simultaneous equations bias (Table 3). Failing to disentangle the (positive) causal effect of \( \text{CO}_2 \) on temperature should make the effect of temperature on \( \text{CO}_2 \) seem stronger and reduce uncertainty about its point estimate. Indeed, as expected, the noninstrumented regressions produce greater feedback estimates with smaller standard errors. While the instrumented univariate regression does produce a similar point estimate and standard error for the coefficient on \( T_t \) as do the instrumented multivariate regressions, it is less useful for estimating millennial feedbacks because it does not allow previous temperature or \( \text{CO}_2 \) concentrations to affect \( T_t \) values.

In estimation of decadal-scale feedbacks, coefficients on the more recent \( \text{CO}_2 \) levels are often significant while the other coefficients are usually not significant (Table 4). This accords with the intuition that, over such short time scales and with the correspondingly small variation in \( \text{CO}_2 \) and temperature over each time step, the time \( t \) \( \text{CO}_2 \) level should be almost wholly determined by the previous period’s \( \text{CO}_2 \) level. Mann et al. [2008] provided several composite temperature records calculated using different instrumental records and combined using different statistical techniques. All results reported in this paper use the reconstruction resulting from their error-in-variables estimation procedure and calibrated using HadCRUT3v instrumental land and

**Table 1.** Estimation Results for the Nondimensional Climate-Carbon Feedback Factor \( f_{cc} \)

<table>
<thead>
<tr>
<th>Time Scale</th>
<th>Specification</th>
<th>Data’s Time Step (years)</th>
<th>( n^a )</th>
<th>( f_{cc} )</th>
<th>s.e.(^b)</th>
<th>( p^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Millennia</td>
<td>Base case</td>
<td>1000</td>
<td>525</td>
<td>0.03</td>
<td>(0.009)</td>
<td>0.0001</td>
</tr>
<tr>
<td>Millennia</td>
<td>Summer insolation</td>
<td>1000</td>
<td>525</td>
<td>0.03</td>
<td>(0.009)</td>
<td>0.0007</td>
</tr>
<tr>
<td>Millennia</td>
<td>Averaged data</td>
<td>1000</td>
<td>536</td>
<td>0.03</td>
<td>(0.01)</td>
<td>0.0001</td>
</tr>
<tr>
<td>Centuries</td>
<td>Base case</td>
<td>1000</td>
<td>525</td>
<td>0.009</td>
<td>(0.01)</td>
<td>0.4</td>
</tr>
<tr>
<td>Centuries</td>
<td>Summer insolation</td>
<td>1000</td>
<td>525</td>
<td>0.006</td>
<td>(0.01)</td>
<td>0.6</td>
</tr>
<tr>
<td>Centuries</td>
<td>Averaged data</td>
<td>1000</td>
<td>536</td>
<td>0.002</td>
<td>(0.02)</td>
<td>0.9</td>
</tr>
<tr>
<td>100 years</td>
<td>–</td>
<td>10</td>
<td>109</td>
<td>0.02</td>
<td>(0.005)</td>
<td>0.003</td>
</tr>
<tr>
<td>100 years</td>
<td>–</td>
<td>25</td>
<td>43</td>
<td>0.01</td>
<td>(0.009)</td>
<td>0.1</td>
</tr>
<tr>
<td>50 years</td>
<td>–</td>
<td>10</td>
<td>109</td>
<td>0.005</td>
<td>(0.002)</td>
<td>0.006</td>
</tr>
<tr>
<td>50 years</td>
<td>–</td>
<td>25</td>
<td>43</td>
<td>0.005</td>
<td>(0.004)</td>
<td>0.2</td>
</tr>
</tbody>
</table>

\(^a\)Number of observations.

\(^b\)Standard errors are robust to arbitrary heteroskedasticity.

\(^c\)Two-tailed \( p \) value for the null hypothesis that \( f_{cc} \) is equal to 0.
ocean hemispheric means. Using the other error-in-variables temperature reconstruction from Mann et al. [2008] does not substantially affect the results, but using the reconstruction developed using the composite plus scale methodology tends to produce estimates that are not significantly different from 0.

5. Discussion

The point estimates and standard errors provide information about the sampling distribution of the mean, but the probability distribution for the feedback factor is more important. Appendix B describes how to develop a probability distribution by extending the hierarchical Bayes framework of Lemoine [2010] to combine this paper’s paleoclimatic estimates. Previous paleoclimatic estimates are converted to feedback form using the factor of 1.2 K (275 ppm)−1 from Torn and Harte [2006] and, in the case of Frank et al. [2010], indicate the range of “likely” values. These previous paleoclimatic estimates assumed that radiative forcing increases linearly with CO2 rather than with the log of CO2.

The posterior distribution implied by the empirical studies is similar to the one implied by coupled models’ output, but considering both types of data together can further constrain the posterior distribution (Figure 5). With only data from coupled models, it is difficult to disentangle the true feedback factor from the biases shared among those models, but empirical estimates provide information about the true feedback factor that is affected by a different set of biases. The posterior distribution resulting from using both types of data has a mean of 0.03 and 5th and 95th percentile values of −0.04 and 0.09. It also indicates a roughly 70% chance that climate-carbon feedbacks are positive, thereby reinforcing other feedbacks such as those due to changes in albedo and water vapor content. Instead of obtaining point estimates and standard errors, future work could develop

![Figure 3.](image)

Figure 3. Estimates of the climate-carbon feedback factor $f_{cc}$. Coupled climate-carbon cycle models are as described by Friedlingstein et al. [2006], and their plotted points are the average of the results from Lemoine [2010] for the three radiative kernels. Error bars show the 95% confidence intervals for this paper’s paleoclimatic estimates. Previous paleoclimatic estimates are converted to feedback form using the factor of 1.2 K (275 ppm)$^{-1}$ from Torn and Harte [2006] and, in the case of Frank et al. [2010], indicate the range of “likely” values. These previous paleoclimatic estimates assumed that radiative forcing increases linearly with CO2 rather than with the log of CO2.

Table 2. Coefficient Estimates and Standard Errors From the Orbital Forcing Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>$n$</th>
<th>$T_1$</th>
<th>$T_{t-1}$</th>
<th>$T_{t-2}$</th>
<th>$C_{t-1}$</th>
<th>Const</th>
<th>$O_t$</th>
<th>$T_{t-1}$</th>
<th>$T_{t-2}$</th>
<th>$C_{t-1}$</th>
<th>Const</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>525</td>
<td>0.005</td>
<td>0.01</td>
<td>−0.01***</td>
<td>0.9***</td>
<td>0.8***</td>
<td>0.007***</td>
<td>1***</td>
<td>−0.2***</td>
<td>2**</td>
<td>−14***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
<td>(0.002)</td>
<td>(0.03)</td>
<td>(0.1)</td>
<td>(0.002)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.8)</td>
<td>(5)</td>
</tr>
<tr>
<td>Summer insolation</td>
<td>525</td>
<td>0.003</td>
<td>0.01</td>
<td>−0.01***</td>
<td>0.9***</td>
<td>0.8***</td>
<td>0.0009***</td>
<td>1***</td>
<td>−0.2***</td>
<td>2**</td>
<td>−14***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.002)</td>
<td>(0.03)</td>
<td>(0.2)</td>
<td>(0.0002)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.8)</td>
<td>(5)</td>
</tr>
<tr>
<td>Averaged data</td>
<td>536</td>
<td>0.001</td>
<td>0.02</td>
<td>−0.01***</td>
<td>0.9***</td>
<td>0.6***</td>
<td>0.005***</td>
<td>1***</td>
<td>−0.4***</td>
<td>0.6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
<td>(0.01)</td>
<td>(0.004)</td>
<td>(0.02)</td>
<td>(0.1)</td>
<td>(0.001)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.6)</td>
<td>(4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specification</th>
<th>$n$</th>
<th>$T_1$</th>
<th>$T_{t-1}$</th>
<th>$T_{t-2}$</th>
<th>$C_{t-1}$</th>
<th>Const</th>
<th>$O_t$</th>
<th>$T_{t-1}$</th>
<th>$T_{t-2}$</th>
<th>$C_{t-1}$</th>
<th>Const</th>
</tr>
</thead>
<tbody>
<tr>
<td>Averaged data</td>
<td>536</td>
<td>0.001</td>
<td>0.02</td>
<td>−0.01***</td>
<td>0.9***</td>
<td>0.6***</td>
<td>0.005***</td>
<td>1***</td>
<td>−0.4***</td>
<td>0.6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
<td>(0.01)</td>
<td>(0.004)</td>
<td>(0.02)</td>
<td>(0.1)</td>
<td>(0.001)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.6)</td>
<td>(4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specification</th>
<th>$n$</th>
<th>$T_1$</th>
<th>$T_{t-1}$</th>
<th>$T_{t-2}$</th>
<th>$C_{t-1}$</th>
<th>Const</th>
<th>$O_t$</th>
<th>$T_{t-1}$</th>
<th>$T_{t-2}$</th>
<th>$C_{t-1}$</th>
<th>Const</th>
</tr>
</thead>
<tbody>
<tr>
<td>Averaged data</td>
<td>536</td>
<td>0.001</td>
<td>0.02</td>
<td>−0.01***</td>
<td>0.9***</td>
<td>0.6***</td>
<td>0.005***</td>
<td>1***</td>
<td>−0.4***</td>
<td>0.6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
<td>(0.01)</td>
<td>(0.004)</td>
<td>(0.02)</td>
<td>(0.1)</td>
<td>(0.001)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.6)</td>
<td>(4)</td>
</tr>
</tbody>
</table>

*Standard errors (in parentheses) are robust to arbitrary heteroskedasticity. Two-tailed $p$ values are for the null hypothesis that the true coefficient is equal to 0: * means $p < 0.1$, ** means $p < 0.05$, and *** means $p < 0.01$; $t$ is in 1000 years.

*Number of observations.

*Units of (ln ppm CO2) K$^{-1}$.

*Units of K (W m$^{-2}$)$^{-1}$.

*Units of K (ln ppm CO2)$^{-1}$.
probability distributions directly from paleoclimatic data and then combine those with coupled models’ predictions. [24] The proper application of this paper’s empirical feedback estimates to anthropogenic climate change depends on the question of interest. Feedback strength may vary with time scale, and future feedbacks will operate in a world with different boundary conditions and with radiative forcing changing with a scale and speed not represented in paleoclimatic data or in data used to tune coupled models. Further, feedback strength may depend on the pace of climate change, and uncertainty about concentration-carbon feedbacks may be more important to the total carbon cycle response than is uncertainty about climate-carbon feedbacks [Gregory et al., 2009]. A complete accounting of carbon cycle uncertainty must include these factors as well as concerns about irreversible changes.

[25] Paleoclimatic records suggest that climate-carbon feedbacks are positive, despite the presence of measurement error that should lead to underestimation of feedback strength. Obtaining more precisely dated paleoclimatic records with denser data could be crucial for better identification of feedback strength, and longer Holocene time series with denser data are important for estimation on subcentury time scales. It appears as if coupled models’ feedback predictions are more apt than are the higher estimates of previous paleoclimatic work. Importantly, combining coupled models’ output with this paper’s empirical estimates sufficiently constrains climate-carbon feedbacks so that they might not be a dominant source of uncertainty about future temperature change. Temperature risk assessments are probably more dominated by the possibility of tipping points and of shared biases among models [O’Neill...

### Table 3. Coefficient Estimates and Standard Errors in Versions of the Base Case Orbital Forcing Specification Without Using Instruments and/or Without Including Lagged Variables as Covariates

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>$T_0$</th>
<th>$T_{-1}$</th>
<th>$T_{-2}$</th>
<th>$C_{-1}$</th>
<th>Const</th>
<th>$O_t$</th>
<th>$f_{cc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Univariate,</td>
<td>638</td>
<td>0.03***</td>
<td>–</td>
<td>–</td>
<td>6***</td>
<td>–</td>
<td>–</td>
<td>0.1***</td>
</tr>
<tr>
<td>noninstrumented</td>
<td></td>
<td>(0.0002)</td>
<td>–</td>
<td>–</td>
<td>(0.008)</td>
<td>–</td>
<td>–</td>
<td>(0.005)</td>
</tr>
<tr>
<td></td>
<td>638</td>
<td>0.007</td>
<td>–</td>
<td>–</td>
<td>5***</td>
<td>0.02**</td>
<td>–</td>
<td>(0.003)</td>
</tr>
<tr>
<td>instrumented</td>
<td></td>
<td>(0.01)</td>
<td>–</td>
<td>–</td>
<td>(0.06)</td>
<td>(0.01)</td>
<td>–</td>
<td>(0.04)</td>
</tr>
<tr>
<td></td>
<td>525</td>
<td>0.02***</td>
<td>–0.0006</td>
<td>–0.009***</td>
<td>0.8***</td>
<td>0.9***</td>
<td>–</td>
<td>0.05***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.03)</td>
<td>(0.1)</td>
<td>–</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>

*a*Standard errors (in parentheses) are robust to arbitrary heteroskedasticity in the multivariate case and also to arbitrary autocorrelation in the univariate cases. Two-tailed p values are for the null hypothesis that the true coefficient is equal to 0. * means $p < 0.1$, ** means $p < 0.05$, and *** means $p < 0.01$. $t$ is in 1000 years. The instrumented univariate case has a robust Kleibergen-Papp F statistic of 6, indicating the potential for a weak instrument problem.

*b*Number of observations.

*c*Units of (ln ppm $CO_2$) K$^{-1}$.

*d*First-stage regression result with units of K (W m$^{-2}$)$^{-1}$.

*e*In the univariate cases, assumes that the coefficient on $T_{-1}$ is certainly equal to zero.
Table 4. Coefficient Estimates and Standard Errors From the Specifications Used to Estimate Decadal-Scale Feedbacks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate 10 Year Time Step (n = 109)</th>
<th>Estimate 100 Year Time Step (n = 43)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆T_i</td>
<td>0.0001** (0.00005)</td>
<td>0.0003 (0.0004)</td>
</tr>
<tr>
<td>∆T_{i-1}</td>
<td>0.00008 (0.00006)</td>
<td>0.0008 (0.0005)</td>
</tr>
<tr>
<td>∆T_{i-2}</td>
<td>0.0002** (0.00009)</td>
<td>0.0006 (0.0005)</td>
</tr>
<tr>
<td>∆T_{i-3}</td>
<td>0.0001* (0.00007)</td>
<td>0.001** (0.0005)</td>
</tr>
<tr>
<td>∆T_{i-4}</td>
<td>0.00004 (0.00007)</td>
<td>-0.0001 (0.0004)</td>
</tr>
<tr>
<td>∆T_{i-5}</td>
<td>0.00002 (0.00001)</td>
<td>0.0005 (0.0003)</td>
</tr>
<tr>
<td>∆T_{i-6}</td>
<td>0.0002* (0.00009)</td>
<td></td>
</tr>
<tr>
<td>∆T_{i-7}</td>
<td>0.0002 (0.0001)</td>
<td></td>
</tr>
<tr>
<td>∆T_{i-8}</td>
<td>0.0002* (0.00008)</td>
<td></td>
</tr>
<tr>
<td>∆T_{i-9}</td>
<td>-0.0002** (0.0001)</td>
<td></td>
</tr>
<tr>
<td>∆T_{i-10}</td>
<td>0.00007 (0.0001)</td>
<td></td>
</tr>
<tr>
<td>∆T_{i-11}</td>
<td>0.00004 (0.00006)</td>
<td></td>
</tr>
<tr>
<td>∆C_{i-1}</td>
<td>1*** (0.1)</td>
<td>1*** (0.3)</td>
</tr>
<tr>
<td>∆C_{i-2}</td>
<td>-0.7*** (0.2)</td>
<td>-0.2 (0.4)</td>
</tr>
<tr>
<td>∆C_{i-3}</td>
<td>0.7** (0.3)</td>
<td>0.2 (0.3)</td>
</tr>
<tr>
<td>∆C_{i-4}</td>
<td>-0.6** (0.2)</td>
<td>-0.2 (0.1)</td>
</tr>
<tr>
<td>∆C_{i-5}</td>
<td>0.4 (0.3)</td>
<td>-0.01 (0.09)</td>
</tr>
<tr>
<td>∆C_{i-6}</td>
<td>-0.3 (0.3)</td>
<td></td>
</tr>
<tr>
<td>∆C_{i-7}</td>
<td>0.3 (0.3)</td>
<td></td>
</tr>
<tr>
<td>∆C_{i-8}</td>
<td>-0.2 (0.2)</td>
<td></td>
</tr>
<tr>
<td>∆C_{i-9}</td>
<td>0.2 (0.2)</td>
<td></td>
</tr>
<tr>
<td>∆C_{i-10}</td>
<td>-0.2* (0.1)</td>
<td></td>
</tr>
<tr>
<td>∆C_{i-11}</td>
<td>0.06 (0.08)</td>
<td></td>
</tr>
</tbody>
</table>

*Standard errors are robust to arbitrary heteroskedasticity. Two-tailed p values are for the null hypothesis that the true coefficient is equal to 0: * means p < 0.1, ** means p < 0.05, and *** means p < 0.01. Coefficients on temperature terms are in units of (ln ppm CO₂) K⁻¹.

and Oppenheimer, 2004; Lenton et al., 2008; Lemoine, 2010]. However, climate policy analyses can be especially sensitive to the positive tail of temperature change distributions because damages may increase nonlinearly with the temperature index and because climate decision-makers are usually modeled as risk averse [Newbold and Daigneault, 2009; Weitzman, 2009]. Because positive climate-carbon feedbacks thicken these policy-relevant positive tails, considering their existence and associated uncertainty is important not just for climate projections but also for economic assessments that may otherwise underestimate climatic risks.

Appendix A: Sources of Bias in Estimating Climate-Carbon Feedbacks

[26] Previous empirical work estimated climate-carbon feedbacks using Little Ice Age data and Vostok ice core data [Scheffer et al., 2006; Torn and Harte, 2006; Cox and Jones, 2008; Frank et al., 2010]. These studies ran univariate ordinary least squares (OLS) regressions of CO₂ on temperature, but the estimates produced by such a regression are vulnerable to several sources of bias that complicate attempts to apply the results to the current global radiative forcing experiment. Adjusting them to use log concentrations, those univariate regressions may be represented as

\[ C_i = \mu + \beta T_i + \epsilon_i. \]  

where \( C_i \) is the log of the CO₂ concentration at time \( t \), \( T_i \) is the temperature at time \( t \), \( \mu \) is a constant term, and \( \epsilon_i \) is the random unobserved error at time \( t \). The parameter of interest is \( \beta \), which ideally gives \( \partial \text{CO}_2 \partial T \) or even \( \partial \text{CO}_2 \partial dT \). The linearized full system may look more like

\[
\begin{align*}
C_i & = \mu + \sum^n_{i=0} \beta_i T_{r-i} + \sum^n_{j=1} \gamma_j C_{r-j} + \eta_i, \\
T_i & = \mu_T + \sum^p_{i=0} \alpha_i C_{t-i} + \sum^q_{j=1} \beta_j T_{t-j} + \nu_i.
\end{align*}
\]  

(A2)

In this representation, CO₂ concentrations and temperature each depend on their own past values, on the past values of the other variable, on the constants \( \mu_C \) and \( \mu_T \), and on the
random errors $\eta_i$ and $\nu_t$. Here, the parameter of interest depends on the allowed time for carbon cycle responses, but it is either $\beta_0$ or some combination of the $\beta$, $\gamma$, $\alpha$, and $\phi$ parameters that gives the effect of a maintained unit change in temperature on future log CO$_2$ concentrations.

[27] Assume for the rest of this section that the parameter of interest is $\beta_0$, which may be the case if the data’s time step is larger than the time scale of interest in the feedback application. When the true system is (A2), estimating $\beta_0$ via the univariate regression in equation (A1) introduces three sources of bias via the correlation between $\epsilon_t$ and $T_t$. First, assume that the true system has, $\forall t > 0$, $\alpha_t = \beta_0 = 0$ and, $\forall t$, $\gamma_t = \phi_t = 0$. In this case, previous CO$_2$ concentrations and previous temperatures would not affect current CO$_2$ concentrations and temperatures, but the current CO$_2$ concentration and the current temperature would affect each other. The simplified system of equations becomes

\[
\begin{align*}
C_t &= \mu_C + \beta_0 T_t + \eta_t \\
T_t &= \mu_T + \alpha_0 C_{t-1} + \nu_t,
\end{align*}
\]  

(A3a)

where $\eta_t = \epsilon_t$ from equation (A1). Let $b_0$ be the OLS estimate of $\beta_0$ from equation (A1) so that \( \text{plim} \ b = \beta_0 + \frac{\text{Cov}(T_t, \epsilon_t)}{\text{Var}(T_t)} \). If $T_t$ is exogenous for $C_t$, then Cov($T_t$, $\epsilon_t$) = 0 and $b$ is a consistent estimator of $\beta_0$. However, from (A3a), Cov($T_t$, $\epsilon_t$) = Cov($T_t$, $\eta_t$) = $\frac{\alpha_0}{1 - \beta_0 \alpha_0} \text{Var}(\epsilon_t)$. Because we know $\alpha_0 > 0$ (indeed, this is the greenhouse effect in this specification), the OLS estimate $b$ is asymptotically biased upward as long as $\epsilon_t$ is uncertain. Unobserved nontemperature factors that affect CO$_2$ levels through $\epsilon_t$ also affect temperature via the usual radiative forcing mechanism, which biases the OLS estimate of the effect of temperature on CO$_2$ by amplifying the relationship between observed temperature and observed CO$_2$. Measurement error in the CO$_2$ data is also subsumed in $\epsilon_t$ and thus can also produce simultaneous equations bias. This bias may be nonexistent if temperature is deemed not to respond to CO$_2$ on the time scale of interest (as Frank et al. [2010] and Scheffer et al. [2006] argued for the Little Ice Age) or if there is both no nontemperature driver of CO$_2$ and no measurement error for CO$_2$. Instrumental variables methods potentially enable one to avoid simultaneous equations bias without making such strong assumptions.

[28] Second, replace the previous paragraph’s assumptions with the assumption that, $\forall t$, $\alpha_t = 0$. This means that CO$_2$ does not affect temperature in the data of interest, which is an explicit reason Scheffer et al. [2006] and Frank et al. [2010] chose to study the Little Ice Age. In addition, assume that $\exists y_0 > 0$ such that $\phi_t \neq 0$. The system of equations now becomes

\[
\begin{align*}
C_t &= \mu_C + \sum_{i=0}^{\infty} \beta_i T_{t-i} + \sum_{j=1}^{\infty} \gamma_j C_{t-j} + \eta_t \\
T_t &= \mu_T + \sum_{j=1}^{\infty} \phi_j T_{t-j} + \nu_t.
\end{align*}
\]  

(A3b)

Simultaneous equations bias does not appear if estimating $\beta_0$ in (A3b) from equation (A1), but the lagged variables create a different problem. In (A1), the error term $\epsilon_t$ is a function of lagged temperature values when the true system is (A3b). However, because previous temperatures affect the temperature observed at time $t$, Cov($T_{t}$, $T_{t-i}$) $\neq 0$ for some $i > 0$, and because previous temperatures also affect CO$_2$ at time $t$ but are omitted from the estimated system (equation (A1)), we have Cov($T_{t}$, $\epsilon_t$) $\neq 0$. The lagged temperatures act as omitted variables that bias estimates of $\beta_0$ in equation (A1). Because these omitted variables are probably positively correlated with $T_t$ and probably have positive coefficients in (A3b), this bias probably also inflates positive estimates of $\beta_0$.

[29] Third, replace the previous paragraphs’ assumptions with the assumption that the true system has, $\forall t > 0$, $\beta_t = 0$ and, $\forall t$, $\alpha_t = \gamma_t = \phi_t = 0$. The true system becomes

\[
\begin{align*}
C_t &= \mu_C + \beta_0 T_t + \eta_t \\
T_t &= \mu_T + \nu_t
\end{align*}
\]  

(A3c)

where $\eta_t$ is uncorrelated with any time’s temperature or with any previous CO$_2$ level. OLS estimation of $\beta_0$ via equation (A1) would be consistent and unbiased with system (A3c) if temperature were measured without error. However, temperature is actually measured but imperfectly. Let the observed temperature values be $T^*_t$, where

\[
T^*_t = T_t + \omega_t,
\]  

(A4)

and $\omega_t$ is a random variable that produces measurement error. Substituting into (A3c), we get

\[
\begin{align*}
C_t^* &= \mu_C + \beta_0 T^*_t + \eta_t^* \\
\eta_t^* &= \eta_t - \beta_0 \omega_t.
\end{align*}
\]  

(A5)

Measurement error $\omega_t$ in $T_t$ induces nonzero correlation between $\eta_t^*$ and the observed $T^*_t$. If $\omega_t$ has variance $\sigma^2_\omega$, we have

\[
\text{Cov}(T^*_t, \eta_t^*) = \text{Cov}(T_t + \omega_t, \eta_t - \beta_0 \omega_t) = -\beta_0 \sigma^2_\omega.
\]  

(A6)

The random, unobserved measurement error in the temperature record biases the OLS estimate of $\beta_0$ toward zero ("attenuation bias"). This measurement error may be due to errors in measurement of isotopes, in inferences about local temperature from isotopes, in inferences about global temperature from local temperature, and in the assignment of relative dates to the recorded temperature and CO$_2$. Measurement error should be the primary source of bias remaining in the present study, and it is to some extent inescapable in work using data from limited paleoclimatic data sets.

Appendix B: Hierarchical Bayes Model for Combining Coupled Models’ Output With Empirical Estimates

[30] This appendix outlines a statistical model which largely follows that described by Lemoine [2010] but is adjusted to include a second group of studies (this paper’s base case paleoclimatic estimates) that may have their own shared biases. Let $f_{j,c}$ represent the true value of the climate-carbon feedback factor and let $\theta_j$ represent the biases shared by group $j$ (where $j$ is an index indicating that studies are coupled models or paleoclimatic estimates). Crucially, assume that $\theta_1$ and $\theta_2$ are independent of each other, meaning that empirical studies’ shared biases are assumed to be independent of those impacting coupled climate-carbon cycle models.

[31] The empirical studies used here are the base case estimate of millennial climate-carbon feedbacks and the
base case estimate of century-scale climate–carbon feedbacks. For each empirical study $i$, $\lambda_{ij}$ represents the divergence between the object of the estimation procedure ($\tilde{z}_i$) and the feedback of interest for projecting future temperature change ($f_{cc}$). $\lambda_{ij}$ includes both the biases idiosyncratic to study $i$ and the biases $\theta_j$ common across empirical studies when applied to future climate change. Here $\lambda_{ij}$ is drawn from a normal distribution centered on its group’s shared biases $\theta_j$ and having standard deviation $\tau_j$. Let $\tilde{z}_i$ be the best estimate for empirical study $i$ with $\tilde{z}_i$ as the standard error of that estimate, where the estimates and standard errors are as reported in the main text.

Finally, for coupled models’ predictions, define $\sigma_j$ to be the standard deviation of a study’s idiosyncratic bias conditional on its shared biases. Each coupled model $i$ generates “observations” of its central feedback estimate $M_{ij}$ by combining its output with a radiative kernel $h$ as described by Soden et al. [2008]. We denote these observations by $y_{hi}$ and let $\phi_j$ be the standard deviation of those observations around $M_{ij}$. The standard deviation $\phi_j$ therefore controls intrastudy variation while $\sigma_j$ controls variation between models. Similarly, $\tau_j$ controls variation between empirical studies while $\tilde{z}_i$ describes variation within a single empirical study’s estimate.

The model can be written as

$$
\lambda_{ij} \sim N(\theta_j, \tau_j) \tag{B1}
$$

$$
\tilde{z}_i \sim t(f_{cc} + \lambda_{ij}, \tilde{z}_i, df) \tag{B2}
$$

$$
M_{ij} \sim N(f_{cc} + \theta_j, \sigma_j) \tag{B3}
$$

$$
y_{hi} \sim N(M_{ij}, \phi_j), \tag{B4}
$$

where $N(\mu, \sigma)$ is a normal distribution with mean $\mu$ and standard deviation $\sigma$ and where $t(x, y, z)$ is a $t$ distribution with location parameter $x$, scale parameter $y$, and shape parameter $z$. $df$ is the models’ degrees of freedom and is
Table B1. Prior Distributions Used for Model Parameters and Plotted in Figure B1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_{c})</td>
<td>({0, 0.15, 2})</td>
</tr>
<tr>
<td>(\theta_{c})</td>
<td>({0, 0.05, 2})</td>
</tr>
<tr>
<td>(\tau_{j})</td>
<td>HC(0.01)</td>
</tr>
<tr>
<td>(\sigma_{j})</td>
<td>HC(0.1)</td>
</tr>
<tr>
<td>(\phi_{j})</td>
<td>HC(0.1)</td>
</tr>
</tbody>
</table>

\(HC(x)\) is a half-Cauchy distribution with scale parameter \(x\), and \(f(x, y, z)\) is a \(f\) distribution with location parameter \(x\), scale parameter \(y\), and shape parameter \(z\). See Lemoine [2010] for more information.

\(^{e}\)Censored so that values are observed to be less than 1.

\(^{f}\)Censored so that values are observed to be between \(-0.5\) and 0.5.

equal to 520 for the base case specifications. The prior distributions are given in Table B1 and plotted in Figure B1, and they follow those used by Lemoine [2010]. The posterior distributions were sampled using Markov chain Monte Carlo methods as implemented in WinBUGS version 1.4.3 [Lunn et al., 2000]. Each posterior distribution generated one million samples after a burn-in period of one million samples. The sample size was large enough for multiple chains to converge on the posterior distributions.

[34] Figure B2 shows the influence of models’ predictions and empirical estimates on the joint distribution for the true feedback factor \(f_{c}\) and the coupled models’ shared bias term \(\theta_{1}\) (where the coupled models are group 1). Data from the coupled models can only constrain the sum \(f_{c} + \theta_{1}\), leading to a ridge in the joint posterior distribution running along values of \(f_{c}\) and \(\theta_{1}\) that produce the same value for \(f_{c} + \theta_{1}\) and have similar prior densities (Figure B2b). However, including the base case empirical results from this paper can further constrain the distribution for \(f_{c}\) because \(\theta_{2}\) is assumed to be independent of \(\theta_{1}\) and the empirical estimates are similar to the coupled models’ predictions. A posterior distribution produced using both types of data still has a ridge along similar values of \(f_{c}\) and \(\theta_{1}\) but the ridge is now shorter because the posterior distribution of \(f_{c}\) is also constrained by the empirical studies’ information about the sum \(f_{c} + \theta_{2}\) (Figure B2c).


References


Lüthi, D., et al. (2008), High-resolution carbon dioxide concentration record 650,000–800,000 years before present, Nature, 453(7193), 397–382, doi:10.1038/nature06949.


D. M. Lemoine, Energy and Resources Group, University of California, Berkeley, 310 Barrows Hall, Berkeley, CA 94720-3050, USA. (dlemoine@berkeley.edu)